Kaon Fragmentation Function and K/π Ratios in Nuclear Collisions

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An improved leading order fragmentation functions set of kaon is proposed based on the experimental data. We compare it with currently available sets, and use it to calculate high-p_T K/π ratios in relativistic proton-proton collisions. A prediction of K/π ratios in relativistic nucleus-nucleus collisions ($d+Au$ and $Au+Au$) at RHIC $\sqrt{s}=200$ GeV is given.

An enhancement of the abundance of strange quarks (s and $\bar{s}$) is expected if a quark-gluon plasma (QGP) is formed in a relativistic nuclear collision, compared to a collision without QGP formation[1]. This asymmetry in the flavor composition of QGP may be reflected in the final particle composition. This is therefore of interest to systematically study the production of strange mesons, in particular kaons, in relativistic proton-proton (pp) and proton-nucleus (pA) collisions. Furthermore, it has been suggested [3] that the measurable K/π ratios are sensitive to the initial density of the QGP in relativistic nucleus-nucleus (AB) collisions. Henceforth, in order to provide a basis for the treatment of kaon production in nuclear collisions at the energies of the Relativistic Heavy Ion Collider (RHIC), it is important and necessary to first develop a good description of kaon production in pp collisions and relying on that, we can then give a reliable studying of hard (high-p_T) K/π probe for nuclear collisions.

In the framework of parton model of perturbative QCD (pQCD), the invariant cross section for inclusive kaon production in a pp collision gets involved with fragmentation functions (FFs), $D_{h/a}(z,Q^2)$, through the factorization theorem [2]. The value of $D_{h/a}(z,Q^2)$ corresponds to the probability for the parton a produced at short distance $\sim 1/Q$ to form a jet that includes the hadron h carrying the fraction z of the longitudinal momentum of parton a. It is a nonperturbative object and hence can not be derived from QCD. During the last few years, the experimental progress at the CERN and the SLAC has motivated people to generate leading-order (LO) and next-to-leading-order (NLO) set of $\pi^{\pm}$, $K^{\pm}$, and $p/\bar{p}$ FFs set [4, 5, 6], which can be utilized in pQCD calculations. However, the available FFs sets for kaon are still not satisfied. For example, in Fig.1, we show the comparison between Kretzer's FFs (K) and that of Kniehl, Kramer, and Pötter (KKP), for the fragmentation process of u quark to $K^{+}+K^{-}$. We find that the differences between KKP-FFs and K-FFs are significant, especially at large-z region.

On the other hand, the experimental data of kaon production in pp collisions in the energy range 20 GeV $\lesssim \sqrt{s} \lesssim$ 40 GeV[7, 8] shows that $K^{+}/\pi^{+} \to 1/2$ at high-p_T region, which reminds us that $D_u^{K^+}/D_u^{\pi^+} \to 1/2$. While interesting enough, it has been implied by Feynman *et al.* that, when $z \to 1$ (z is the longitudinal momentum fraction of a parton), $D_u^{K^+}/D_u^{\pi^+} \to (1-\beta)/\beta$, in which β is a constant parameter in Feynman-Field parameterization[9, 10]. In current letter, based on the KKP-FFs of pion and experimental data, we propose the following improved LO FFs (Z) $D_Z^{K^{\pm}}$ of kaon,

$$D_{a,Z}^{K^{\pm}} = \frac{1}{2} D_{a,KKP}^{\pi^{\pm}} \ , \tag{1}$$

in which $D_{a,KKP}^{\pi^{\pm}}$ is the corresponding LO KKP-FFs of pion and parton a can be the $u(\bar{u})$, $d(\bar{d})$, $s(\bar{s})$ quark or gluon (g).

To deal with charged kaons, i.e., K^{+} and K^{-}, we now consider the scheme to separate the above FFs for kaon. A natural way to do is as follows: we assume, say, for u valence quark of $K^{+}(u\bar{s})$,

$$D_{u,Z}^{K^{+}} = \frac{1}{2} D_{u,KKP}^{\pi^{+}} \ , \tag{2}$$

so that we have

$$D_{u,Z}^{K^{-}} = 0 \ , \tag{3}$$

which means the contribution of u valence quark to $K^{-}(\bar{u}s)$ is vanished.

Starting with eqn-s.(2) and (3), we compare our improved Z-FFs with KKP-FFs and K-FFs in Fig.1. We can see that the Z-FFs agrees with K-FFs in the most of moderate z region, but disagrees with it at large-z region (with $z > 0.7$), where in stead tends to agree with KKP-FFs.

Based on this Z-FFs set, the LO invariant cross section of kaon production in pp collisions is described in the pQCD-improved parton model on the basis of the factorization theorem as a convolution [10]:

$$\begin{aligned} E_K \frac{d\sigma_{pp}^{LO}}{d^3p} = & \sum_{abcd} \int dx_a \, dx_b \, d\mathbf{k}_{Ta} \, d\mathbf{k}_{Tb} \, dz_c \\ & \times g_{a/p}(\mathbf{k}_{Ta}) \, f_{a/p}(x_a,Q^2) \, g_{b/p}(\mathbf{k}_{Tb}) \, f_{b/p}(x_b,Q^2) \\ & \times \frac{d\sigma^{LO}}{d\hat{t}}(ab\to cd) \frac{D_{K/c}^{Z}(z_c,Q'^2)}{\pi z_c^2} \hat{s}\delta(\hat{s}+\hat{t}+\hat{u}) \, , \end{aligned} \tag{4}$$

where $f_{a/p}(x,Q^2)$ and $f_{b/p}(x,Q^2)$ are the parton distribution functions (PDFs) for the colliding partons a and b in the interacting protons as functions of momentum fraction x, at scale Q, $d\sigma/d\hat{t}$ is the hard scattering cross

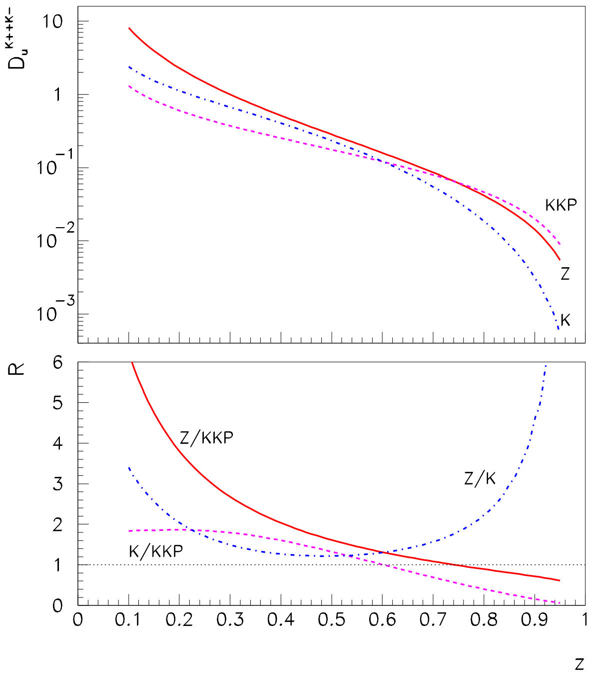

FIG. 1: The comparisons of improved FFs (full line) with KKP-FFs (dashed line) and K-FFs (dot-dashed line). In top panel we show the fragmentation function of u quark to $K^+ + K^-$, $D_u^{K^+ + K^-}$. In the bottom panel, the ratios between different FFs sets, Z/KKP (full line), K/KKP (dashed line), and Z/K (dot-dashed line), are shown, as a function of z at fixed scale $Q^2 = 25\ GeV^2$.

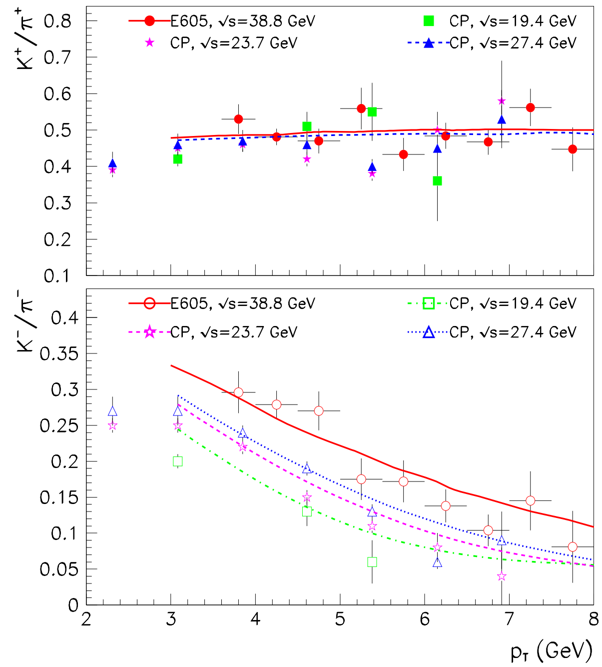

FIG. 2: The comparison of experimental (symbols) and pQCD-calculated (curves) K/π ratios: K^+/π^+ (top panel) and K^-/π^- (bottom panel), as a function of p_T at $\sqrt{s}$ = 19.4, 23.7, 27.4, and 38.8 GeV, respectively, in pp collisions. In top panel, we only show K^+/π^+ ratios at $\sqrt{s} = 27.4$ (full line) and 38.8 (dashed line) GeV. The kaon production is calculated based on improved Z-FFs. The experimental data of K/π ratios are from Refs.[7, 8].

section of the partonic subprocess $ab \to cd$, and the FFs, $D^Z_{K/c}(z_c, Q'^2)$ gives the probability for parton c to fragment into kaon with momentum fraction z_c at scale Q'. The superscript "Z" is used here for our improved Z-FFs. We use the convention that the parton-level Mandelstam variables are written with a 'hat' (like $\hat{t}$ above). The scales are fixed in the present work as $Q = p_T/2$ and $Q' = p_T/2z_c$. The function $g_{a/p}(\mathbf{k}_T)$ is the intrinsic transverse momentum distributions of parton a inside nucleon. We follow the phenomenological approach in the present work, taking $g(\mathbf{k}_T)$ to be a Gaussian:

$$g(\mathbf{k}_T) \ = \frac{1}{\pi\langle k_T^2\rangle} e^{-k_T^2/\langle k_T^2\rangle} \ . \tag{5}$$

Here $\langle k_T^2\rangle$ is the 2-dimensional width of the k_T distribution and it is related to the magnitude of the average transverse momentum of one parton as $\langle k_T^2\rangle = 4\langle k_T\rangle^2/\pi$.

In this investigation we use LO partonic cross sections, together with LO PDFs (GRV) [11]. The Monte-Carlo integrals are carried out by the VEGAS-routine [12].

We analysed experimental data of inclusive K production in pp collisions in the center-of-mass energy range 20 GeV $\lesssim \sqrt{s} \lesssim$ 40 GeV[7, 8]. We find a good agreement between our model and experimental data on both K^+/π^+ and K^-/π^-. In our analysis, the intrinsic transverse momentum distribution of parton, $\langle k_T^2\rangle$ = 1.7, 2.0, 2.2, and 2.2 GeV^2, are used for both kaon and pion at $\sqrt{s}$ = 19.4, 23.7, 27.4, and 38.8 GeV, respectively [13].

High-p_T kaon production data begun to emerge from RHIC, at center-of-mass energies $\sqrt{s}$ = 130 and 200 GeV [14, 15]. This provides us with the opportunity to predict kaon production in $d + Au$ and $Au + Au$ collisions at higher energies. For nucleus-nucleus collisions, the Glauber model[16] is the standard description. The LO inclusive kaon production in AB collisions can be written in the following formlism:

$$E_K \frac{d\sigma^{LO}_{AB}}{d^3p} = \sum_{abcd} \int d^2b d^2r\, T_A(r)\, T_B(|\mathbf{b}-\mathbf{r}|) \int dx_a$$
$$\times\, dx_b d\mathbf{k}_{Ta} d\mathbf{k}_{Tb}\, dz_c\ g_{a/A}(\mathbf{k}_{Ta}) f_{a/A}(x_a, Q^2)$$

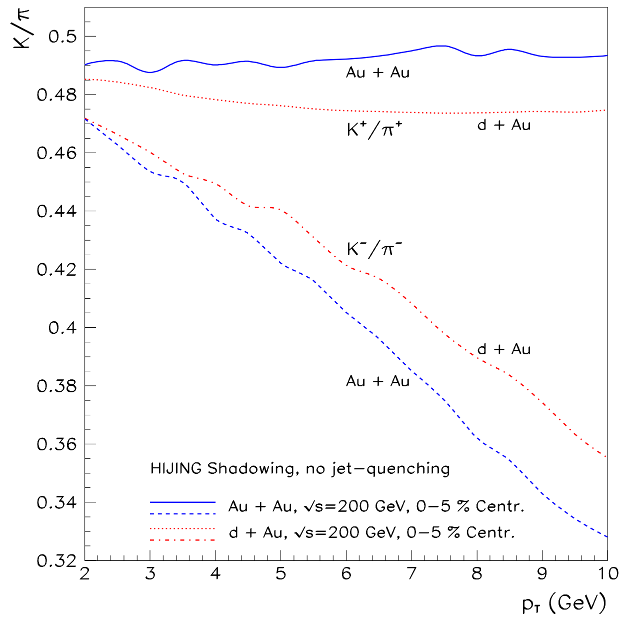

FIG. 3: The predicted K/π ratios for $d + Au$ and $Au + Au$ collisions at RHIC energy $\sqrt{s} = 200$ GeV (0 - 5% centrality) with HIJING shadowing, but without jet quenching. The values of $\langle k_T^2 \rangle_{pp} = 1.7\ GeV^2$, $\nu_m = 4$, and $C = 0.4$ GeV^2 are fixed [13] in the calculation.

$$\times\, g_{b/B}(\mathbf{k}_{Tb})\, f_{b/B}(x_b, Q^2)\, \frac{d\sigma^{LO}}{d\hat{t}}(ab \to cd) \times \frac{D^Z_{K/c}(z_c, Q'^2)}{\pi z_c^2} \hat{s}\delta(\hat{s} + \hat{t} + \hat{u}). \quad (6)$$

The predicton on K/π ratios at RHIC energy, $\sqrt{s} = 200$ GeV, for both $d + Au$ and $Au + Au$ collisions, are shown in Fig.3.

In above calculations, we include parton multiscattering due to the effect of nuclear medium. The extra contribution to the width can be related to the number of nucleon-nucleon (NN) collisions in the medium as follows:

$$\langle k_T^2 \rangle_{pA} = \langle k_T^2 \rangle_{pp} + C \cdot h_{pA}^{sat}(b)\ . \quad (7)$$

Here $\langle k_T^2 \rangle_{pp}$ is the width of the transverse momentum distribution of partons in pp collisions and $\langle k_T^2 \rangle_{pp} = 1.7$ GeV^2 at $\sqrt{s} = 200$ GeV [13]. The function $h_{pA}(b)$ describes the number of *effective* NN collisions at impact parameter b which impart an average transverse momentum squared C and it can be written in terms of the number of collisions suffered by the incoming proton in the target nucleus, $\nu_A(b) = \sigma_{NN} t_A(b)$, where σ_{NN} is the inelastic NN cross section. In this work, we take the 'saturated' prescription [13, 17]., i.e., we assume that only one associated NN collision is responsible for the $\langle k_T^2 \rangle$ enhancement, so $h_{pA}^{sat}(b)$ was equated to a maximum value ν_m, whenever $\nu_A(b) \geq \nu_m = 2$. We fix the value of $\nu_m = 4$ and an associated value of $C = 0.4$ GeV^2[13].

we also approximately include shadowing effect (the PDFs are modified due to nuclear environment) [18, 19] and the isospin asymmetry of heavy nuclei into the nuclear PDFs. We use the average nuclear dependence and the scale-independent parameterization of the HIJING shadowing function [18],

$$f_{a/A}(x, Q^2) = R_{a/A}(x)\left[\frac{Z}{A} f_{a/p}(x, Q^2)\right] + R_{a/A}(x) \times \left[\left(1 - \frac{Z}{A}\right) f_{a/n}(x, Q^2)\right], \quad (8)$$

where $f_{a/n}(x, Q^2)$ is the PDF for the neutron.

In conclusion, an improved LO FFs set of kaon production is proposed in present letter, based on the experimental data. Relying on this improved FFs set, we calculate the high-p_T K/π ratios in pp collisions in center-of-mass energy range 20 GeV $\lesssim \sqrt{s} \lesssim$ 40 GeV, and we find our results in pp collisions are in good agreement with experiment data. We also predict K/π ratios for $d + Au$ and $Au + Au$ collisions at $\sqrt{s} = 200$ GeV to provide background information for further comparison with the high-p_T kaon production data of RHIC. The future systematic investigation of kaon production with this improved FFs set in pp, pA and AB collisions will be very interesting and be reported in other place. For AB collisions with the consideration of jet quenching, the current FFs set of kaon will give very important insight about the existence and properties of the QGP.

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